MC02 COLLISIONS & 2D CONSERVATION

SPH4U

EQUATIONS

- Kinetic Energy of Elastic Collisions

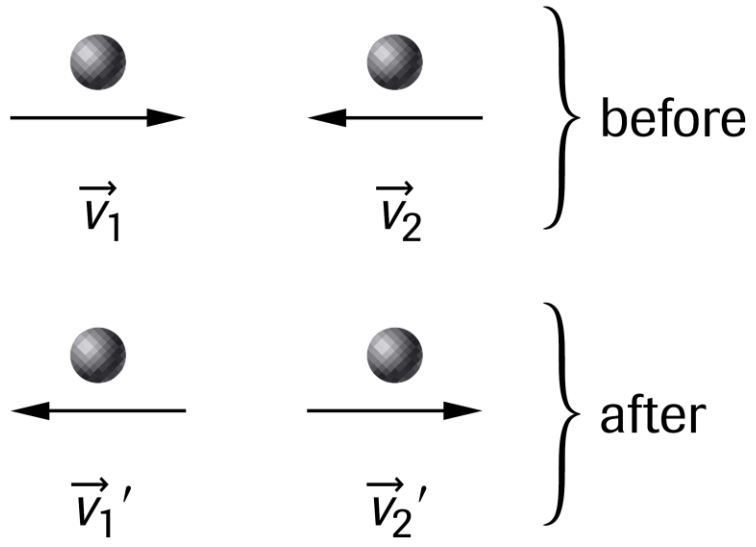
$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2$$

ELASTIC COLLISIONS

- **Elastic Collision:** a collision in which the total kinetic energy after the collision equals the total kinetic energy before the collision

$$E_K' = E_K$$
$$\vec{p}\,' = \vec{p}$$

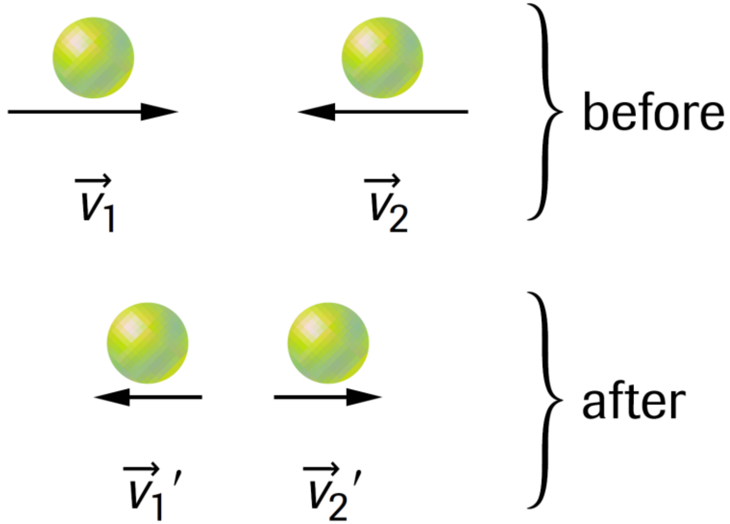
  - IE: superballs



INELASTIC COLLISIONS

- **Inelastic Collision:** a collision in which the total kinetic energy after the collision is different from the total kinetic energy before the collision

$$E_K' \neq E_K$$
$$\vec{p}\,' = \vec{p}$$

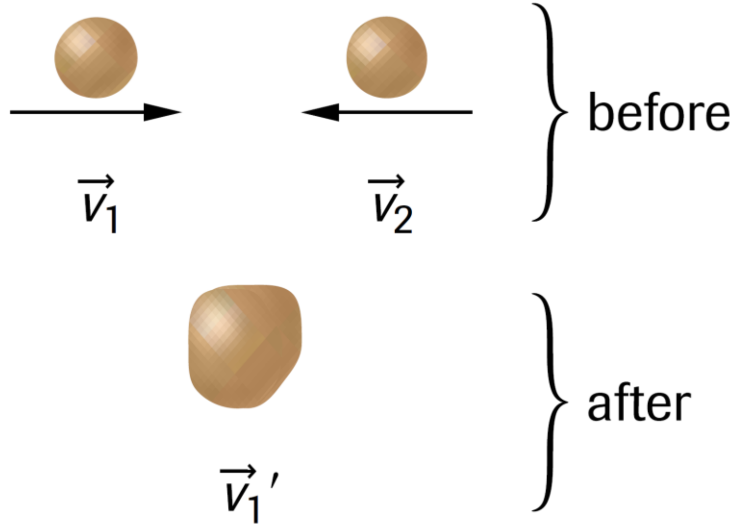


  - IE: tennis balls

COMPLETELY INELASTIC COLLISIONS

- **Completely Inelastic Collision:** a collision in which there is a maximum decrease in kinetic energy after the collision since the <u>objects stick together and move at the same velocity</u>

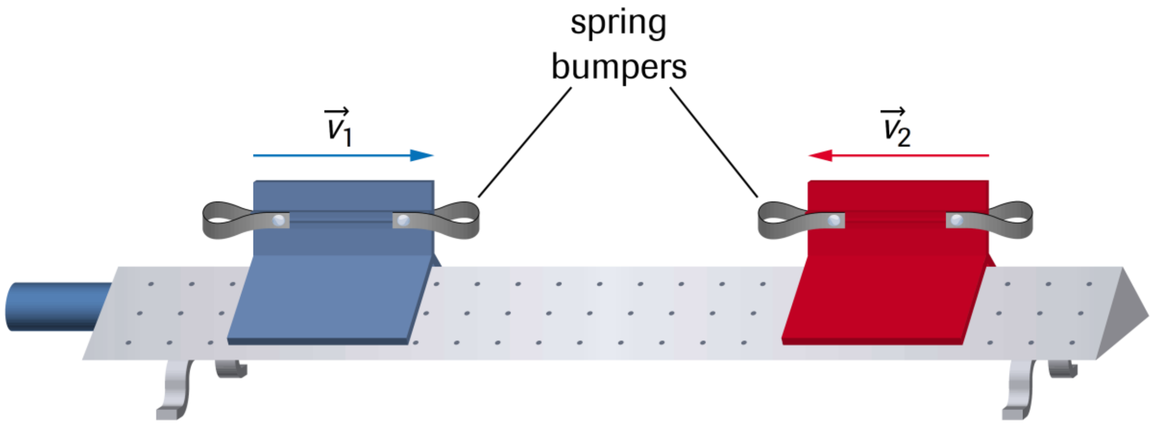
$$E_K' < E_K$$
$$\vec{p}\,' = \vec{p}$$



  - IE: balls of soft putty

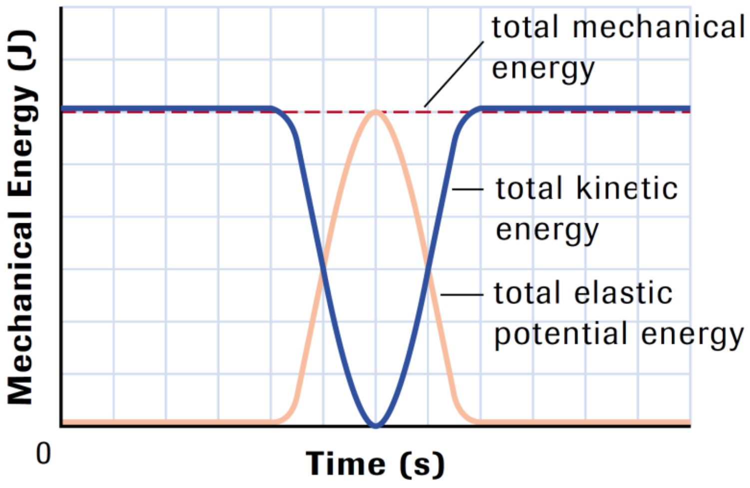
COLLISIONS

- We compare the kinetic energies of objects before and after a collision
- During the collision, kinetic energy is transferred to elastic potential energy
  - $E_e = max$ when $E_K = 0$
- The elastic potential energy is then transferred back into kinetic, dropping back to zero when the objects no longer touch



NOTES ABOUT COLLISIONS

- We compare the kinetic energies of objects before and after a collision
- Elastic Collisions: energy is transferred between E_e and E_K
  - $E_T = E_K + E_e$
- Inelastic Collisions: kinetic energy lost from the system is converted into other forms of energy
  - thermal, sound, light, etc.



SOLVING COLLISION PROBLEMS

- For all types of collisions, momentum is conserved

$$\vec{p} = \vec{p}'$$

$$mv_1 + mv_2 = mv_1' + mv_2'$$

- Elastic Collisions: kinetic energy is also conserved

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2$$

PROBLEM 1

A billiard ball with mass m and initial speed v_1, undergoes a head-on elastic collision with another billiard ball, initially stationary, with the same mass m. What are the final speeds of the two balls?

PROBLEM 1 – SOLUTIONS

Figure 8 shows the initial and final diagrams. We choose the $+x$-axis as the direction of motion of the initially moving ball (ball 1). Since the problem states that the collision is elastic, we know that the total initial kinetic energy equals the total final kinetic energy. Momentum is conserved in this collision. We can thus write two equations, one for kinetic energy and one for momentum:

$$\frac{1}{2}mv_1^{\,2} + \frac{1}{2}mv_2^{\,2} = \frac{1}{2}mv_1'^{\,2} + \frac{1}{2}mv_2'^{\,2}$$

$$mv_1 + mv_2 = mv_1' + mv_2'$$

where the subscript 1 refers to the initially moving ball and the subscript 2 refers to the initially stationary ball. Note that the v's represent velocity components (not velocity magnitudes) and can be positive or negative. Since the masses are equal, they cancel:

$$v_1^{\,2} + v_2^{\,2} = v_1'^{\,2} + v_2'^{\,2}$$

$$v_1 + v_2 = v_1' + v_2'$$

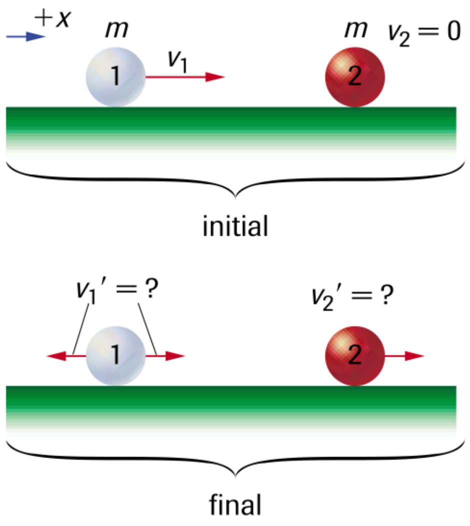


Figure 8

PROBLEM 1 – SOLUTIONS CONT.

Since ball 2 is initially stationary, $v_2 = 0$, and we can write:

$$\begin{aligned} {v_1}^2 &= {v_1'}^2 + {v_2'}^2 \\ v_1 &= v_1' + v_2' \end{aligned}$$

We now have two equations and two unknowns, so we rearrange the latter equation to solve for v_1':

$$v_1' = v_1 - v_2'$$

Substituting for v_1':

$$\begin{aligned} {v_1}^2 &= (v_1 - v_2')^2 + {v_2'}^2 \\ &= {v_1}^2 - 2v_1v_2' + {v_2'}^2 + {v_2'}^2 \\ 0 &= -2v_1v_2' + 2{v_2'}^2 \\ 0 &= -v_2'(v_1 - v_2') \end{aligned}$$

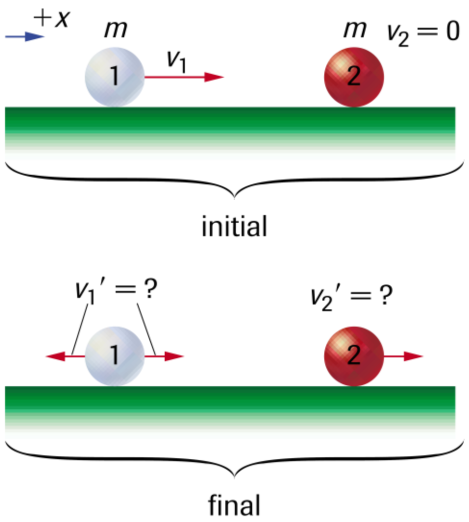


Figure 8

PROBLEM 1 – SOLUTIONS CONT.

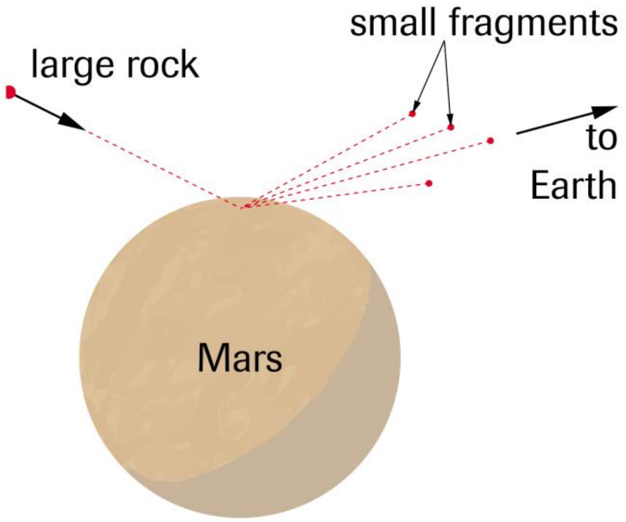
Therefore, either $v_2' = 0$ (which is not an appropriate solution since it means that no collision occurred) or $v_1 - v_2' = 0$. Thus, we can conclude that $v_2' = v_1$. Substituting this value into the equation $v_1 = v_1' + v_2'$:

$$\begin{aligned} v_1 &= v_1' + v_1 \\ v_1' &= 0 \end{aligned}$$

Therefore, ball 1, which was initially moving, is at rest after the collision ($v_1' = 0$); ball 2, which was initially stationary, has the same speed after the collision that ball 1 had before the collision ($v_2' = v_1$). Note that this conclusion is not valid for all elastic collisions in which one object is initially stationary—the colliding objects must have the same mass.

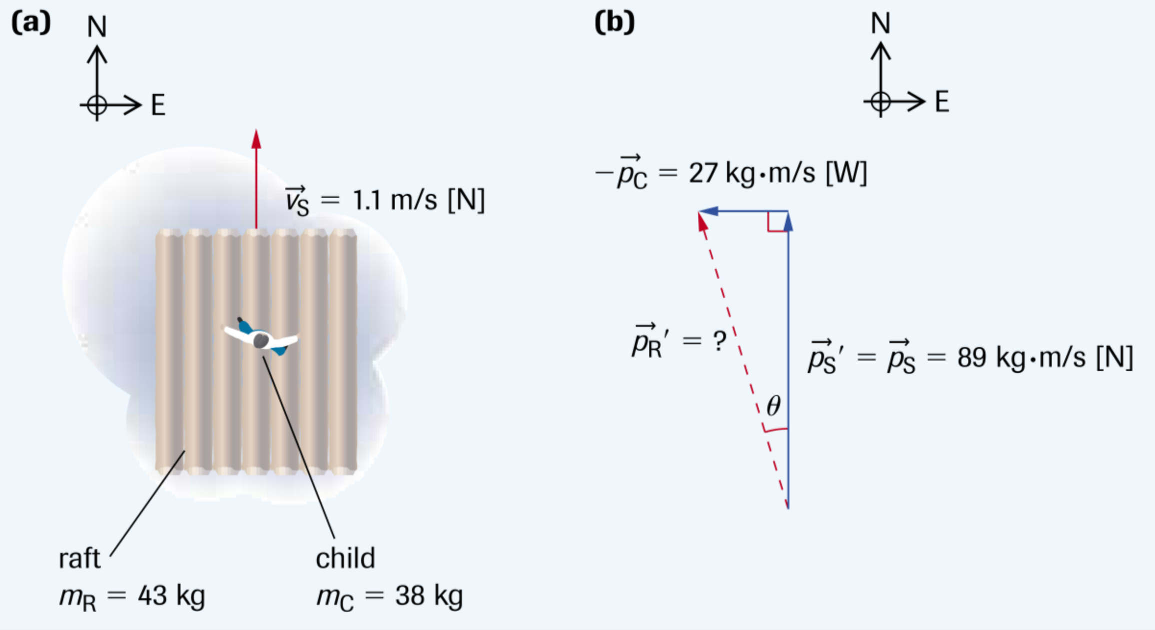
CONSERVATION OF MOMENTUM IN 2 DIMENSIONS

- Since both net force and momentum are vectors, we can look at the components of those vectors
  - If $\vec{p} = \vec{p}'$, then $p_x = p'_x$ and $p_y = p'_y$
- The law of conservation of momentum applies to any system with zero net force
  - all types of collisions
  - ejection of gases to control spacecraft, etc.



PROBLEM 2

A 38-kg child is standing on a 43-kg raft that is drifting with a velocity of 1.1 m/s [N] relative to the water. The child then walks on the raft with a net velocity of 0.71 m/s [E] relative to the water. Fluid friction between the raft and water is negligible. Determine the resulting velocity of the raft relative to the water.



PROBLEM 2 – SOLUTIONS

Figure 2(a) shows the situation. Since there is no net force acting on the system, momentum is conserved. Thus,

$$\vec{p}_S = \vec{p}_S{}'$$

where the subscript S represents the system. Finding the initial momentum of the system:

$m_S = 38 \text{ kg} + 43 \text{ kg} = 81 \text{ kg}$

$\vec{v}_S = 1.1 \text{ m/s [N]}$

$\vec{p}_S = ?$

$$\begin{aligned}\vec{p}_S &= m_S\vec{v}_S \\ &= (81 \text{ kg})(1.1 \text{ m/s [N]}) \\ \vec{p}_S &= 89 \text{ kg}\cdot\text{m/s [N]}\end{aligned}$$

PROBLEM 2 – SOLUTIONS CONT.

The final momentum of the system is equal to the vector addition of the child (indicated by subscript C) and the raft (indicated by subscript R):

$$\vec{p}_S' = \vec{p}_C' + \vec{p}_R'$$

Determine $\vec{p}_C'$:

$m_C = 38 \text{ kg}$

$\vec{v}_C' = 0.71 \text{ m/s [E]}$

$\vec{p}_C' = ?$

$$\begin{aligned} \vec{p}_C' &= m_C \vec{v}_C' \\ &= (38 \text{ kg})(0.71 \text{ m/s [E]}) \\ \vec{p}_C' &= 27 \text{ kg}\cdot\text{m/s [E]} \end{aligned}$$

PROBLEM 2 – SOLUTIONS CONT.

Since $\vec{p}_S = \vec{p}_S'$, we can now solve for $\vec{p}_R'$:

$$\vec{p}_R' = \vec{p}_S' - \vec{p}_C'$$
$$\vec{p}_R' = \vec{p}_S' + (-\vec{p}_C')$$

Figure 2(b) shows the vector subtraction. Using the law of Pythagoras, we find:

$$\begin{aligned} |\vec{p}_R'|^2 &= |\vec{p}_S'|^2 + |\vec{p}_C'|^2 \\ |\vec{p}_R'| &= \sqrt{(89\ \text{kg}\cdot\text{m/s})^2 + (27\ \text{kg}\cdot\text{m/s})^2} \\ |\vec{p}_R'| &= 93\ \text{kg}\cdot\text{m/s} \end{aligned}$$

The angle θ can now be found:

$$\begin{aligned} \theta &= \tan^{-1}\frac{27\ \text{kg}\cdot\text{m/s}}{89\ \text{kg}\cdot\text{m/s}} \\ \theta &= 17^\circ \end{aligned}$$

PROBLEM 2 – SOLUTIONS CONT.

Thus, the direction of the raft's final momentum and final velocity is 17° W of N.

Finally, we solve for the final velocity of the raft:

$$\vec{p}_R' = m_R \vec{v}_R'$$

$$\vec{v}_R' = \frac{\vec{p}_R'}{m_R}$$

$$= \frac{93\ \text{kg}\cdot\text{m/s}\ [17^\circ\ \text{W of N}]}{43\ \text{kg}}$$

$$\vec{v}_R' = 2.2\ \text{m/s}\ [17^\circ\ \text{W of N}]$$

The resulting velocity of the raft relative to the water is 2.2 m/s [17° W of N].

SUMMARY – ELASTIC AND INELASTIC COLLISIONS

- In all elastic, inelastic, and completely inelastic collisions involving an isolated system, the momentum is conserved.
- In an elastic collision, the total kinetic energy after the collision equals the total kinetic energy before the collision.
- In an inelastic collision, the total kinetic energy after the collision is different from the total kinetic energy before the collision.
- In a completely inelastic collision, the objects stick together and move with the same velocity, and the decrease in total kinetic energy is at a maximum.
- Elastic collisions can be analyzed by applying both the conservation of kinetic energy and the conservation of momentum simultaneously.

SUMMARY – CONSERVATION OF MOMENTUM IN 2 DIMENSIONS

- Collisions in two dimensions are analyzed using the same principles as collisions in one dimension: conservation of momentum for all collisions for which the net force on the system is zero, and both conservation of momentum and conservation of kinetic energy if the collision is elastic.

PRACTICE

Readings

- Section 5.3, pg 246
- Section 5.4, pg 254

Questions

- pg 253 #2-6
- pg 258 #1-4